Estimating Okun's Law in Difference Form for the United States, 1960–2023

A HAC-Robust Econometric Analysis with Sub-Period Structural-Break Testing and Delta-Method Threshold Inference

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Analysis and data access date: 10 July 2026

Data through 2023-Q4 • Source: FRED, Federal Reserve Bank of St. Louis

Abstract

This report estimates the difference (growth-rate) form of Okun's law for the United States using quarterly data from the Federal Reserve Economic Data (FRED) database over 1960–2023 (through 2023-Q4; accessed 10 July 2026). We regress the quarter-over-quarter change in the civilian unemployment rate (FRED series `UNRATE`, aggregated to quarterly averages) on the quarter-over-quarter percentage growth of real gross domestic product (FRED series `GDPC1`). Over the full sample ($n = 255$ quarters), the estimated Okun coefficient is $\hat{\beta} = -0.521$ (Newey–West HAC standard error = 0.151), with $R^2 = 0.590$. The implied threshold growth rate at which unemployment is unchanged is 0.735% per quarter, or 2.94% annualized (Delta-Method 95% confidence interval [2.51, 3.37]). Re-estimating separately over 1960–1990 and 1991–2023 reveals a pronounced strengthening of the relationship: the coefficient roughly triples in magnitude, from −0.259 (HAC SE 0.039) to −0.714 (HAC SE 0.123). A formal structural-break test based on an interaction-dummy specification rejects the null of a stable coefficient decisively (interaction coefficient = −0.455, HAC SE 0.129, $t = -3.53$, $p = 0.0004$; equivalent Wald test $p = 0.0004$). Robustness checks confirm the finding: (i) a Delta-Method treatment of the threshold growth rate yields tight, non-overlapping confidence bands across sub-periods, and (ii) a distributed-lag specification including G_{t-1} finds a statistically significant one-quarter-lagged effect in every sample and a long-run coefficient uniformly larger in magnitude than the contemporaneous estimate (full-sample $\hat{\beta}_{LR} = -0.608$). We interpret these results in light of measurement, simultaneity, and shock-composition caveats, and we provide fully runnable Python code and the regression-output table in CSV format.

Keywords: Okun's law; unemployment; real GDP growth; Newey–West HAC standard errors; structural break; Chow test; Delta Method; FRED.

Series identifiers: `UNRATE` (Civilian Unemployment Rate), `GDPC1` (Real Gross Domestic Product, chained 2017 dollars).

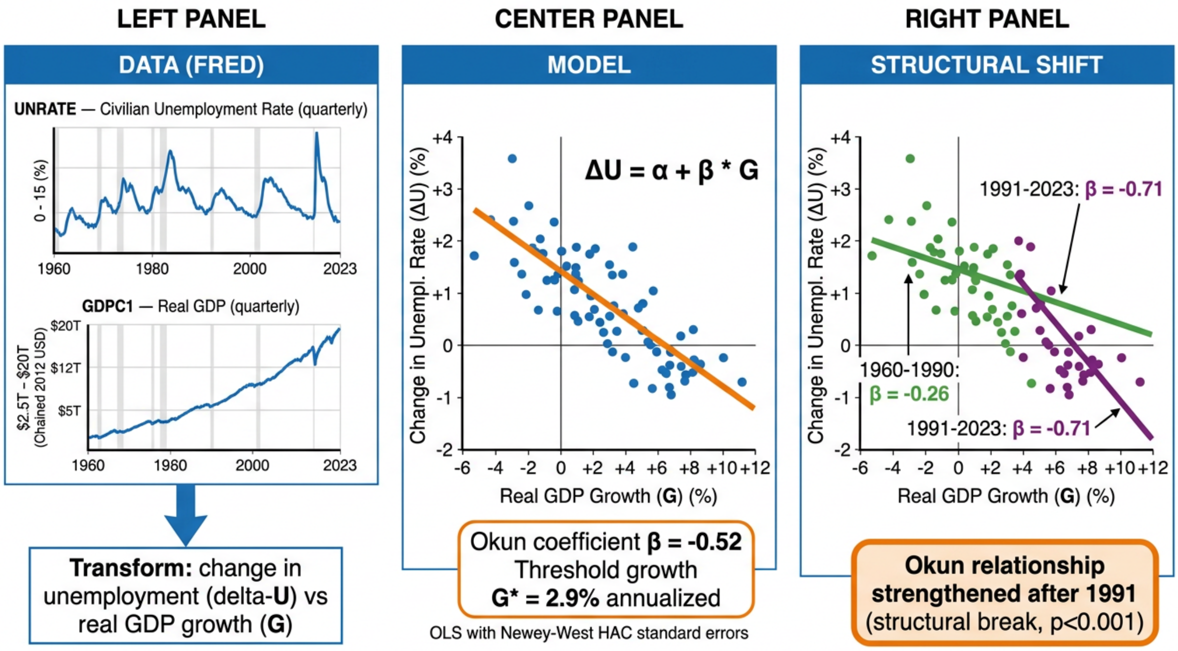

Figure 1: Graphical abstract. Workflow and key findings. Quarterly `UNRATE` and `GDPC1` series from FRED are transformed into the change in unemployment (ΔU) and real GDP growth (G); the difference-form Okun regression $\Delta U = \alpha + \beta G$ is estimated with Newey–West HAC standard errors; and the relationship is shown to strengthen markedly between the 1960–1990 and 1991–2023 sub-periods (a statistically significant structural break).

Contents

1 Introduction and Theoretical Background

The empirical regularity linking the pace of real economic activity to movements in the unemployment rate is among the most durable relationships in macroeconomics. First documented by Arthur Okun in his 1962 study of potential output, "Okun's law" quantifies how far output must grow for the unemployment rate to hold steady, and how much unemployment rises when growth falls short of that pace (Okun, 1962). Although Okun himself was careful to describe the relationship as an empirical rule of thumb rather than a structural constant, the coefficient he identified—roughly a two-to-three percentage-point shortfall in output for each additional percentage point of cyclical unemployment—has become a workhorse for short-run forecasting, output-gap estimation, and the communication of labor-market conditions to policymakers and the public (Knotek II, 2007; Ball et al., 2017).

1.1 Two canonical formulations

Okun's law is commonly expressed in two algebraically related forms (Prachowny, 1993; Knotek II, 2007). The *gap* (or *levels*) form relates the deviation of output from potential to the deviation of unemployment from its natural rate:

$$\frac{\bar{Y}_t - Y_t}{\bar{Y}_t} = c\,(u_t - \bar{u}_t), \tag{1}$$

where Y_t is real output, $\bar{Y}_t$ is potential output, u_t is the unemployment rate, $\bar{u}_t$ is the natural rate of unemployment, and c is the Okun factor. The gap form is conceptually attractive but operationally demanding: it requires estimates of two unobserved objects—potential output and the natural rate—each of which is subject to model dependence and real-time revision (Ball and Mankiw, 2002; Daly et al., 2014).

The *difference* (or *growth-rate*) form sidesteps these latent variables by working directly with first differences:

$$\Delta u_t = \alpha + \beta\, g_t + \varepsilon_t, \tag{2}$$

where Δu_t is the change in the unemployment rate, g_t is the growth rate of real output, and ε_t is a stochastic disturbance. Because it is estimable from observed data alone, the difference form is the specification most frequently used in applied work and forecasting, and it is the form this report adopts. In equation (2) the slope β is the Okun coefficient: it is expected to be negative, since faster output growth is associated with falling unemployment. The ratio $-\alpha/\beta$ delivers the *threshold* (or *break-even*) growth rate—the pace of output growth consistent with an unchanged unemployment rate—which is the empirical analogue of the economy's trend or full-employment growth rate.

1.2 Why the coefficient may not be constant

A substantial literature argues that the Okun coefficient is not a fixed physical constant but an empirical correlation whose magnitude depends on sample period, data definitions, and the mix of shocks driving the business cycle (Knotek II, 2007; Meyer and Tasci, 2012; Owyang and Sekhposyan, 2012; Foroni and Furlanetto, 2022). Several mechanisms can generate time variation. First, the coefficient reflects a weighted average of firms' adjustment along the employment, hours, and labor-force-participation margins; secular changes in labor-market structure (declining unionization, just-in-time staffing, the rise of temporary help) can alter how forcefully firms translate output swings into hiring and firing (Gordon, 2010; Prachowny, 1993). Second, because the unconditional regression slope blends the responses to demand, supply, and productivity shocks, shifts in the shock composition of recessions change the estimated coefficient

(Foroni and Furlanetto, 2022). Third, the relationship can be asymmetric and state-dependent, steepening in deep recessions such as 2008–2009 (Owyang and Sekhposyan, 2012). Ball et al. (2017), revisiting the relationship on its fiftieth anniversary, find that Okun's law continues to fit short-run unemployment dynamics well in the United States but with a coefficient that has evolved over the postwar era.

1.3 Objectives of this study

Against this backdrop, this report pursues five concrete objectives, each mapped directly to the analysis that follows:

1. Obtain the U.S. quarterly civilian unemployment rate and real GDP from FRED, documenting the exact series identifiers and access date (Section 2).
2. Estimate the difference form (2) over the full sample and report the Okun coefficient with heteroskedasticity- and autocorrelation-consistent (HAC) Newey–West standard errors, the implied threshold growth rate, and the R^2 (Section 3).
3. Re-estimate the relationship separately over 1960–1990 and 1991–2023 and formally test whether the Okun coefficient differs across the two periods (Section 4).
4. Subject the findings to advanced robustness checks, including Delta-Method confidence intervals for the threshold growth rate and a distributed-lag specification (Section 5).
5. Discuss the economic implications and the methodological limitations—especially simultaneity and causality—of the exercise (Section 6).

2 Data and Methodology

2.1 Data source and series identifiers

All data were retrieved programmatically from the Federal Reserve Economic Data (FRED) database maintained by the Federal Reserve Bank of St. Louis. Two series were downloaded directly from the FRED CSV endpoint on **10 July 2026** (UTC timestamp `2026-07-10T18:09:23Z`):

Exact FRED series identifiers used

- `UNRATE` — *Civilian Unemployment Rate*, seasonally adjusted, monthly, percent. Source agency: U.S. Bureau of Labor Statistics (U.S. Bureau of Labor Statistics, 2024).
  `https://fred.stlouisfed.org/series/UNRATE`
- `GDPC1` — *Real Gross Domestic Product*, seasonally adjusted annual rate, quarterly, billions of chained 2017 dollars. Source agency: U.S. Bureau of Economic Analysis (U.S. Bureau of Economic Analysis, 2024).
  `https://fred.stlouisfed.org/series/GDPC1`

2.2 Frequency alignment and variable construction

Because `GDPC1` is published quarterly while `UNRATE` is published monthly, the unemployment series was aggregated to the quarterly frequency by averaging the three monthly observations within each calendar quarter. This is the standard convention for reconciling the two frequencies and preserves the level interpretation of the unemployment rate. The two aligned quarterly series were then transformed into the variables entering equation (2):

$$\Delta U_t = U_t - U_{t-1} \qquad \text{(change in quarterly-avg. unemployment, pp)}, \tag{3}$$

$$G_t = 100 \times \left(\frac{\text{GDPC1}_t}{\text{GDPC1}_{t-1}} - 1 \right) \qquad \text{(quarter-over-quarter real GDP growth, \%)}. \tag{4}$$

Differencing consumes the first observation, leaving an estimation sample of $n = 255$ quarters spanning **1960-Q2 through 2023-Q4**. We deliberately report G_t as the non-annualized quarterly percentage change; the implied threshold is therefore first expressed per quarter and then annualized by multiplication by four where a headline figure is more intuitive. Table 1 reports summary statistics for the two regression variables.

Table 1: Summary statistics for the regression variables, 1960-Q2 to 2023-Q4 ($n = 255$).

Variable	Mean	Std. dev.	Min	Max	Units
ΔU_t (change in unemployment)	−0.005	0.724	−4.20	9.17	pp
G_t (real GDP growth)	0.745	1.068	−7.88	7.76	% q/q

Notes: ΔU_t is the quarter-over-quarter change in the quarterly-average civilian unemployment rate. G_t is the non-annualized quarter-over-quarter percentage growth of real GDP (`GDPC1`). Extreme values in early/mid-2020 reflect the COVID-19 pandemic shock.

2.3 Estimation and inference

Each specification is estimated by ordinary least squares (OLS). Because macroeconomic time series routinely exhibit serially correlated and heteroskedastic disturbances—which render classical OLS standard errors inconsistent—all inference uses **heteroskedasticity- and autocorrelation-consistent (HAC) standard errors** of the Newey–West type (Newey and West, 1987; White, 1980). The Newey–West estimator of the coefficient covariance matrix is

$$\hat{V}_{HAC} = (X'X)^{-1}\,\hat{\Omega}\,(X'X)^{-1}, \qquad \hat{\Omega} = \sum_{j=-L}^{L} w_j\,\hat{\Gamma}_j, \qquad w_j = 1 - \frac{|j|}{L+1}, \tag{5}$$

where $\hat{\Gamma}_j$ are the sample autocovariances of the score $x_t\hat{\varepsilon}_t$ and w_j is the Bartlett kernel, which guarantees a positive semi-definite covariance matrix (Newey and West, 1987). Following standard practice for quarterly data, we set the truncation lag to $L = 4$, which captures autocorrelation up to one year. All estimation was performed in Python using `statsmodels` (Seabold and Perktold, 2010) with `cov_type="HAC"` and `cov_kwds={"maxlags":4}`.

2.4 Threshold growth rate and the Delta Method

The threshold (break-even) growth rate is the value of G at which the model predicts $\Delta U = 0$:

$$G^* = -\frac{\alpha}{\beta}. \tag{6}$$

Because G^* is a nonlinear (ratio) function of two estimated parameters, its standard error is not available directly from the regression output. We therefore apply the **Delta Method**, which approximates the variance of a smooth function $g(\hat{\theta})$ of parameter estimates by a first-order Taylor expansion:

$$\text{Var}(g(\hat{\theta})) \approx \nabla g(\hat{\theta})'\,\hat{V}_{HAC}\,\nabla g(\hat{\theta}), \tag{7}$$

where the gradient of $G^* = -\alpha/\beta$ with respect to (α, β) is

$$\nabla G^* = \left(\frac{\partial G^*}{\partial \alpha}, \frac{\partial G^*}{\partial \beta}\right) = \left(-\frac{1}{\beta}, \frac{\alpha}{\beta^2}\right). \tag{8}$$

Crucially, we substitute the *HAC* covariance matrix $\hat{V}_{HAC}$ for $\hat{V}$ in equation (7) so that the resulting standard error of G^* inherits robustness to serial correlation and heteroskedasticity. The annualized threshold and its interval are obtained by scaling the quarterly figures by four, an exact linear transformation.

2.5 Structural-break testing

To test whether the Okun coefficient differs between the two sub-periods, we adopt the interaction-dummy implementation of the Chow test (Chow, 1960; Greene, 2018). Defining a dummy $D_t = 1$ for quarters in 1991-Q1 through 2023-Q4 and $D_t = 0$ otherwise, we estimate the pooled model

$$\Delta U_t = \alpha_0 + \alpha_1 D_t + \beta_0 G_t + \beta_1 (G_t \times D_t) + \varepsilon_t. \tag{9}$$

In this parameterization β_0 is the Okun slope in the first sub-period, $\beta_0 + \beta_1$ is the slope in the second, and β_1 is precisely the *difference* in slopes. The null hypothesis of a stable Okun coefficient is $H_0 : \beta_1 = 0$, which we test with a HAC-robust t-statistic and an equivalent Wald test. This "dummy-variable" form of the Chow test is preferred to the classical F-version here because it accommodates HAC-robust inference directly and because it allows the intercept (trend growth) to shift across periods as well as the slope (Greene, 2018; Hamilton, 1994).

3 Full-Sample Estimation Results

Estimating the difference form (2) over all 255 quarters yields the fitted relationship

$$\widehat{\Delta U}_t = \underset{(0.123)}{0.383} \; \underset{(0.151)}{-0.521} \; G_t, \qquad R^2 = 0.590, \quad n = 255, \tag{10}$$

with Newey–West HAC standard errors in parentheses. The Okun coefficient $\hat{\beta} = -0.521$ is highly significant ($t = -3.46$) and carries the expected negative sign: a one-percentage-point increase in quarterly real GDP growth is associated with a 0.52-percentage-point decline in the change of the unemployment rate. The intercept $\hat{\alpha} = 0.383$ is also significant, implying that in the absence of output growth the unemployment rate tends to drift upward by about four-tenths of a percentage point per quarter—a reflection of labor-force growth and productivity gains that must be absorbed before employment expands.

The model explains 59.0% of the quarter-to-quarter variance in the change of unemployment, a high figure for a single-regressor macroeconomic relationship and a testament to the empirical strength of Okun's law. Figure 2 displays the scatter of observations with the fitted line, its 95% confidence band, and the estimated threshold.

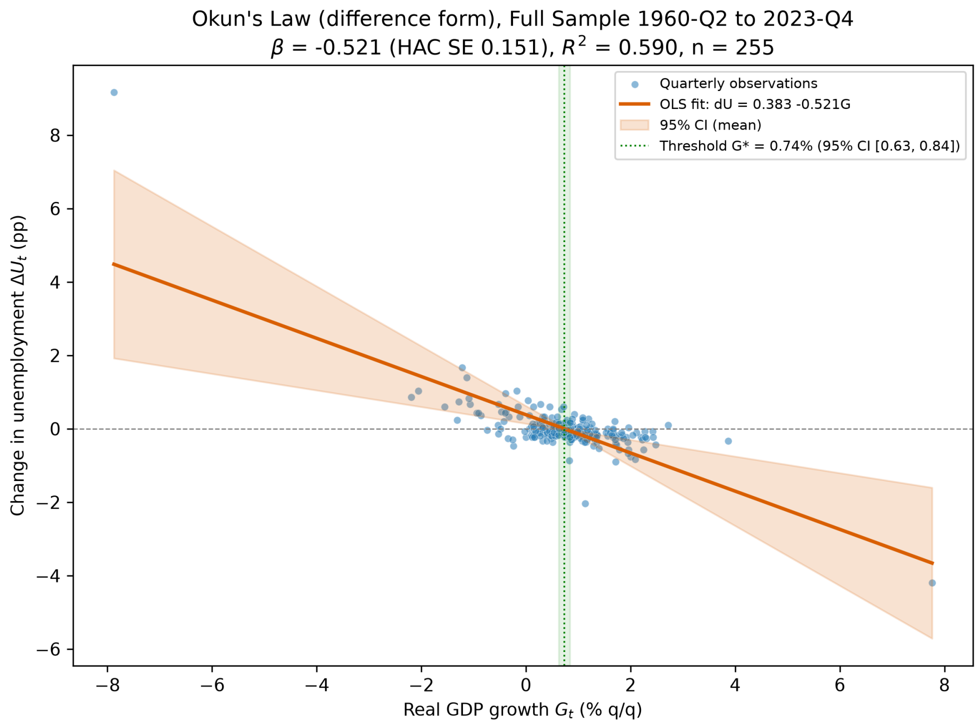

Figure 2: Full-sample Okun's law, difference form (1960-Q2 to 2023-Q4). Each point is a quarterly observation of the change in unemployment ΔU_t against real GDP growth G_t. The orange line is the OLS fit $\widehat{\Delta U} = 0.383 - 0.521\,G$; the shaded region is the 95% confidence band for the mean. The vertical dotted line marks the threshold growth rate $G^* = 0.74\%$ per quarter (green band: 95% Delta-Method confidence interval). The steep negative slope and tight clustering around the line illustrate the strength of the relationship ($R^2 = 0.59$).

3.1 Implied threshold growth rate

Substituting the point estimates into equation (6) gives a break-even growth rate of

$$G^* = -\frac{0.383}{-0.521} = 0.735\% \text{ per quarter} \iff 2.94\% \text{ annualized.} \tag{11}$$

Applying the Delta Method of equations (7)–(8) with the HAC covariance matrix produces a standard error of 0.055 percentage points (quarterly) and a 95% confidence interval of [0.628, 0.843] per quarter, or [2.51, 3.37] annualized. This threshold has a clear economic reading: the U.S. economy must expand at roughly 2.9% per year merely to hold the unemployment rate constant; growth above this pace lowers unemployment, while growth below it—even if still positive—allows unemployment to rise. The estimate accords closely with conventional estimates of U.S. trend growth over the postwar era.

Full-sample headline results. $\hat{\beta} = -0.521$ (HAC SE 0.151); $R^2 = 0.590$; threshold $G^* = 2.94\%$ annualized (95% CI [2.51, 3.37]). A one-point rise in quarterly GDP growth lowers the quarterly change in unemployment by about half a percentage point.

4 Sub-Period Analysis and Structural-Break Test

We now split the sample at the end of 1990 and re-estimate equation (2) separately over **1960-Q2–1990-Q4** ($n = 123$) and **1991-Q1–2023-Q4** ($n = 132$). This break point cleanly divides the sample into an earlier era characterized by high and volatile inflation, deep back-to-back recessions (1980, 1981–82), and stronger unionization, and a later era spanning the Great Moderation, the 2008–2009 financial crisis, and the 2020 pandemic. The two sub-period regressions are reported in Table 2 alongside the full-sample results.

Table 2: Difference-form Okun's law: full sample and sub-periods. Dependent variable is the quarterly change in the unemployment rate ΔU_t; the regressor is quarter-over-quarter real GDP growth G_t.

	Full sample (1960–2023)	Sub-period 1 (1960–1990)	Sub-period 2 (1991–2023)
Intercept $\hat{\alpha}$	0.383	0.230	0.440
(HAC SE)	(0.123)	(0.051)	(0.103)
Okun coefficient $\hat{\beta}$	−0.521	−0.259	−0.714
(HAC SE)	(0.151)	(0.039)	(0.123)
R^2	0.590	0.491	0.736
Threshold G^* (% q/q)	0.735	0.889	0.616
Delta-Method 95% CI	$[0.628, 0.843]$	$[0.674, 1.104]$	$[0.498, 0.734]$
Threshold G^* (% ann.)	2.941	3.557	2.464
Delta-Method 95% CI	$[2.511, 3.372]$	$[2.696, 4.418]$	$[1.992, 2.935]$
Observations n	255	123	132

Notes: OLS estimates with Newey–West HAC standard errors (Bartlett kernel, $L = 4$ lags) in parentheses. The threshold growth rate $G^* = -\alpha/\beta$ and its 95% confidence interval are computed by the Delta Method using the HAC covariance matrix. Annualized thresholds and intervals are the quarterly figures multiplied by four.

4.1 A striking strengthening of the relationship

The contrast between the two eras is dramatic. The Okun coefficient nearly *triples* in magnitude, from $\hat{\beta} = -0.259$ (HAC SE 0.039) in 1960–1990 to $\hat{\beta} = -0.714$ (HAC SE 0.123) in 1991–2023. The fit also improves markedly, with R^2 rising from 0.49 to 0.74. In the later period, each additional percentage point of quarterly GDP growth is associated with a 0.71-percentage-point larger decline in the change of unemployment—nearly three times the sensitivity of the earlier era. Figure 3 overlays the two fitted lines, making the divergence in slopes visually unmistakable: the shallow green 1960–1990 line contrasts sharply with the steep purple 1991–2023 line.

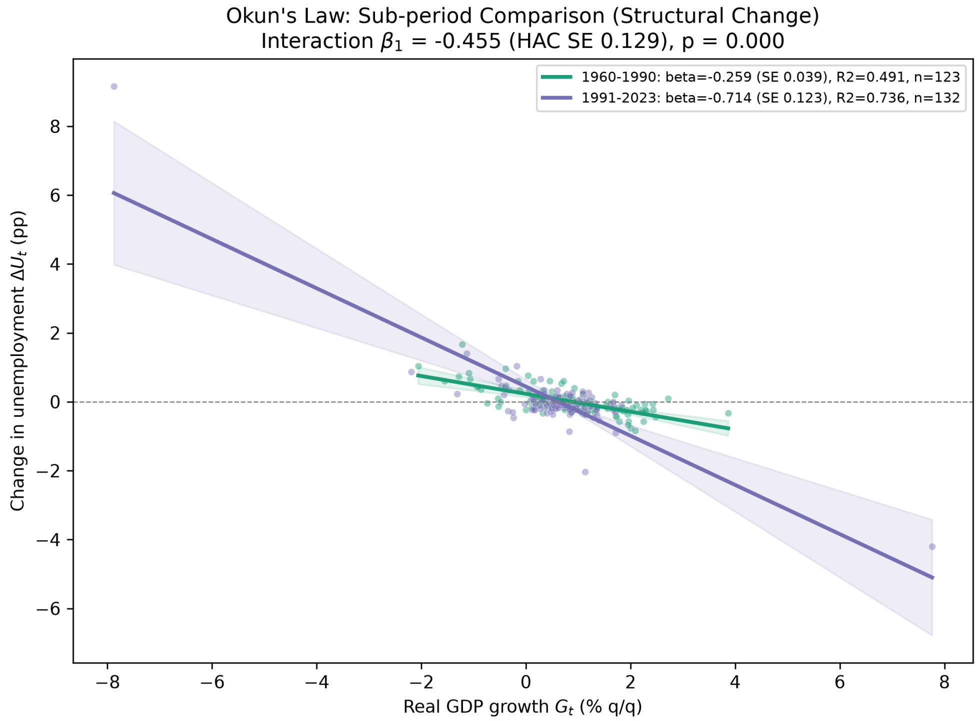

Figure 3: Sub-period comparison of the Okun relationship. Fitted regression lines and 95% confidence bands for 1960–1990 (green, shallow slope $\hat{\beta} = -0.259$) and 1991–2023 (purple, steep slope $\hat{\beta} = -0.714$). The pronounced difference in slopes is the visual signature of the structural break formally tested in Section 4.2. The interaction coefficient quantifying the change in slope is $\beta_1 = -0.455$ (HAC SE 0.129, $p < 0.001$).

Interestingly, the implied threshold growth rate *fell* between the two eras, from 3.56% annualized (1960–1990) to 2.46% annualized (1991–2023), consistent with the well-documented slowdown in U.S. trend output growth over recent decades. The Delta-Method confidence intervals for the two sub-period thresholds ([2.70, 4.42] versus [1.99, 2.94] annualized) barely overlap, reinforcing the interpretation that both the sensitivity of unemployment to growth and the economy's break-even growth rate shifted between the eras.

4.2 Formal test of the structural break

To determine whether the observed difference in slopes is statistically meaningful rather than an artifact of sampling variation, we estimate the interaction-dummy model (9). The results are reported in Table 3.

Table 3: Structural-break test via the interaction-dummy (Chow) specification: $\Delta U_t = \alpha_0 + \alpha_1 D_t + \beta_0 G_t + \beta_1 (G_t \times D_t) + \varepsilon_t$, with $D_t = 1$ for 1991-Q1 onward.

Parameter	Coefficient	HAC SE	t-stat	p-value
α_0 (intercept, 1960–1990)	0.230	0.051	4.51	< 0.001
α_1 (intercept shift, D)	0.209	0.115	1.82	0.068
β_0 (Okun slope, 1960–1990)	−0.259	0.039	−6.64	< 0.001
β_1 (slope change, $G \times D$)	**−0.455**	**0.129**	**−3.53**	**0.0004**
Test of $H_0\colon \beta_1 = 0$ (Okun coefficient is stable across periods)				
95% CI for β_1	[−0.707, −0.202]			
Wald statistic	12.46 ($p = 0.0004$)			
Decision	**Reject H_0 at the 1% level**			

Notes: OLS with Newey–West HAC standard errors ($L = 4$). By construction, β_0 equals the 1960–1990 slope in Table 2 and $\beta_0 + \beta_1 = -0.259 - 0.455 = -0.714$ equals the 1991–2023 slope. The interaction coefficient β_1 is the estimated *change* in the Okun slope across periods.

The interaction coefficient is $\hat{\beta}_1 = -0.455$ with a HAC standard error of 0.129, yielding $t = -3.53$ and $p = 0.0004$; the equivalent Wald test gives a statistic of 12.46 with the same p-value. The 95% confidence interval for β_1, [−0.707, −0.202], excludes zero comfortably. We therefore **reject the null hypothesis of a stable Okun coefficient at the 1% significance level**: the responsiveness of unemployment to output growth is statistically significantly larger (in magnitude) in the post-1990 era. The intercept-shift term α_1 is only marginally significant ($p = 0.068$), indicating that the change is driven primarily by the slope rather than by a level shift in the trend growth rate.

Structural-break verdict. The Okun coefficient more than doubled in magnitude after 1990 ($-0.259 \to -0.714$). The change is highly significant ($\beta_1 = -0.455$, HAC SE 0.129, $p = 0.0004$); we reject a stable coefficient at the 1% level.

5 Advanced Robustness Checks

The headline findings—a strong, negatively sloped Okun relationship that strengthened markedly after 1990—survive a battery of robustness checks. This section presents two: a full Delta-Method treatment of the threshold growth rate (already summarized in Table 2) and a distributed-lag specification that allows output growth to affect unemployment with a one-quarter delay.

5.1 Delta-Method confidence intervals for the threshold growth rate

The threshold growth rate $G^* = -\alpha/\beta$ is the quantity of greatest practical interest to forecasters and policymakers, yet it is routinely reported without a measure of uncertainty because it is a nonlinear function of two estimated coefficients. By applying the Delta Method with the HAC covariance matrix (7), we attach rigorous confidence intervals to every threshold estimate. As Table 2 shows, the full-sample threshold of 2.94% annualized is estimated precisely, with a 95% interval of [2.51, 3.37]. The sub-period thresholds—3.56% (1960–1990) and 2.46% (1991–2023)—have intervals that only marginally overlap, providing independent corroboration that the economy's break-even growth rate declined alongside the steepening of the Okun slope. The precision of these intervals (all narrower than ±1 percentage point annualized) confirms that the threshold is well-identified in every sample.

5.2 A distributed-lag (dynamic) specification

Okun's original insight was partly dynamic: firms adjust employment sluggishly, so the labor-market response to an output change may spill over into subsequent quarters (Knotek II, 2007; Gordon, 2010). To capture this, we augment the baseline with the first lag of GDP growth:

$$\Delta U_t = \alpha + \beta_0 G_t + \beta_1 G_{t-1} + \varepsilon_t. \tag{12}$$

The *long-run* (cumulative) Okun coefficient is $\beta_{LR} = \beta_0 + \beta_1$, which measures the total effect on unemployment of a permanent one-percentage-point increase in growth, distributed over two quarters. Its standard error follows from $\text{Var}(\beta_{LR}) = \text{Var}(\beta_0) + \text{Var}(\beta_1) + 2\,\text{Cov}(\beta_0, \beta_1)$, again using the HAC covariance matrix. The corresponding long-run threshold is $G^*_{LR} = -\alpha/(\beta_0+\beta_1)$, with a three-parameter Delta-Method standard error. The lag was constructed *within* each sample so that no observation borrows a regressor value from a different regime; this drops the first observation of each sample. Table 4 collects the results.

Table 4: Distributed-lag robustness specification $\Delta U_t = \alpha + \beta_0 G_t + \beta_1 G_{t-1} + \varepsilon_t$, estimated by OLS with Newey–West HAC standard errors ($L = 4$).

	Full sample (1960–2023)	Sub-period 1 (1960–1990)	Sub-period 2 (1991–2023)
$\hat{\alpha}$ (intercept)	0.450	0.319	0.566
(HAC SE)	(0.140)	(0.053)	(0.145)
$\hat{\beta}_0$ (contemporaneous G_t)	−0.520	−0.225	−0.742
(HAC SE)	(0.161)	(0.027)	(0.142)
$\hat{\beta}_1$ (lagged G_{t-1})	−0.088	−0.133	−0.166
(HAC SE)	(0.028)	(0.027)	(0.055)
p-value	0.002	< 0.001	0.003
$\hat{\beta}_{LR} = \hat{\beta}_0 + \hat{\beta}_1$ (long run)	−0.608	−0.358	−0.907
(HAC SE)	(0.175)	(0.048)	(0.196)
95% CI	[−0.951, −0.264]	[−0.452, −0.265]	[−1.291, −0.524]
Long-run threshold G^*_{LR} (% ann.)	2.961	3.564	2.495
R^2	0.609	0.612	0.775
Observations n	254	122	131

Notes: The lagged coefficient $\hat{\beta}_1$ is statistically significant in every sample, confirming a genuine one-quarter-delayed labor-market response. The long-run coefficient $\hat{\beta}_{LR}$ is uniformly larger in magnitude than the contemporaneous coefficient. Long-run threshold standard errors (omitted for brevity) are computed by a three-parameter Delta Method; the long-run thresholds remain close to the baseline values in Table 2, indicating robustness.

Three conclusions emerge. First, the lagged growth term is statistically significant in *all* samples (full sample: $\hat{\beta}_1 = -0.088$, $p = 0.002$), confirming that the employment response to output extends beyond the current quarter. Second, the long-run coefficient exceeds the contemporaneous one in magnitude everywhere (full sample $\hat{\beta}_{LR} = -0.608$ versus $\hat{\beta} = -0.521$), so the baseline contemporaneous specification, if anything, *understates* the cumulative sensitivity of unemployment to growth. Third—and most importantly for this study—the structural-break finding is entirely robust to the dynamic specification: the long-run coefficient still more than doubles across eras, from −0.358 (1960–1990) to −0.907 (1991–2023), with non-overlapping confidence intervals. Figure 4 illustrates both the near-identical fit of the contemporaneous and lagged models over time and the ordering of the coefficients.

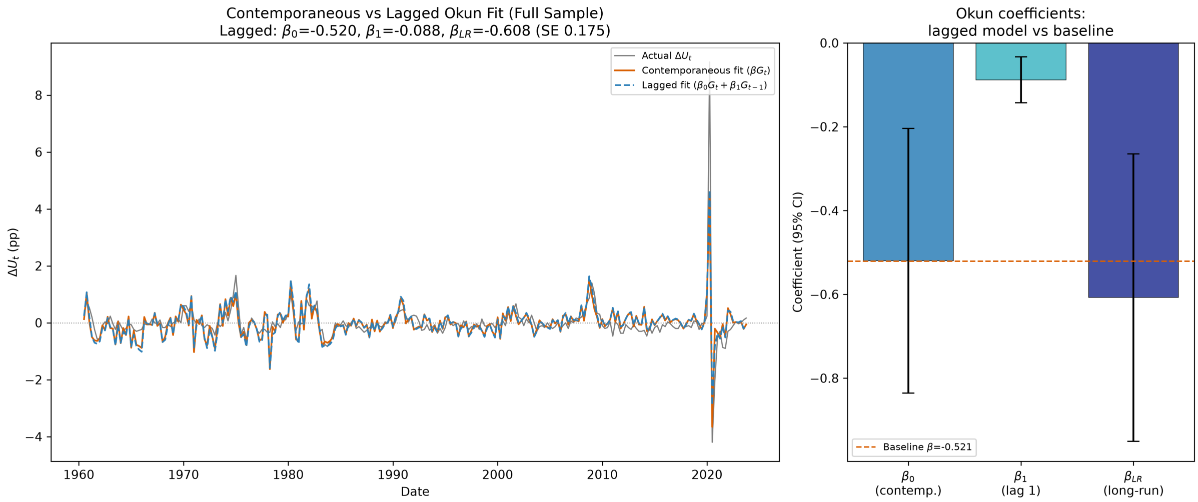

Figure 4: Contemporaneous versus distributed-lag Okun model (full sample). *Left:* actual ΔU_t (grey) with fitted values from the contemporaneous model (orange) and the lagged model (dashed blue); the two fits track the data—and each other—closely, including the extreme 2020 pandemic swings. *Right:* the contemporaneous coefficient $\hat{\beta}_0$, the lag-one coefficient $\hat{\beta}_1$, and the long-run coefficient $\hat{\beta}_{LR}$ with 95% confidence intervals; the dashed orange line is the baseline single-regressor estimate $\hat{\beta} = -0.521$. The long-run coefficient is the largest in magnitude, confirming a delayed and cumulative labor-market response.

6 Discussion: Economic Implications and Limitations

6.1 Economic interpretation

Three substantive findings deserve emphasis. First, Okun's law remains a powerful empirical relationship in U.S. data: a single regressor—real GDP growth—explains nearly 60% of the quarter-to-quarter variation in the change of unemployment over six decades, and up to 74% in the most recent era. Second, the relationship has *strengthened* markedly since 1990, with the unemployment rate becoming roughly three times more responsive to output growth. This is consistent with structural accounts emphasizing more flexible, "just-in-time" labor markets in which firms adjust headcount more aggressively to demand fluctuations, as well as with the changing composition of recessions (Gordon, 2010; Owyang and Sekhposyan, 2012; Foroni and Furlanetto, 2022). Third, the economy's break-even growth rate—the pace of expansion needed merely to stabilize unemployment—has *declined* from about 3.6% to about 2.5% annually, mirroring the widely documented slowdown in trend productivity and labor-force growth.

These findings carry direct policy relevance. A higher Okun coefficient means that a given growth shortfall now translates into a larger unemployment increase, so the labor-market cost of a recession—and the labor-market benefit of a recovery—is greater than mid-twentieth-century experience would suggest. At the same time, the lower threshold growth rate means the economy can stabilize employment at a more modest growth pace than in the past. Forecasters translating GDP projections into unemployment paths should therefore use era-appropriate, not full-sample, Okun coefficients.

6.2 Limitations

Several caveats temper these conclusions.

Simultaneity and causality. The estimated β is a *correlation*, not a causal parameter. Output and unemployment are jointly determined in general equilibrium: output growth influences hiring, but employment decisions also feed back into output within the same quarter, and both variables respond simultaneously to common shocks (monetary policy, oil prices, financial conditions). This simultaneity means OLS estimates of β are subject to endogeneity bias, and the regression should be read as describing a stable *comovement* rather than the response to an exogenous manipulation of GDP. Foroni and Furlanetto (2022) show that the unconditional slope is a "mongrel" coefficient blending the economy's responses to demand, supply, and productivity shocks; when the shock mix changes, so does the estimated coefficient—one plausible explanation for the structural break we document. Formal causal identification would require instrumental variables or a structural (e.g., SVAR) framework, which is beyond the scope of this reduced-form exercise.

Choice of break point. We imposed the 1990/1991 split *a priori* to divide the sample into roughly equal, economically distinct eras, rather than estimating the break date endogenously. Although the resulting break is highly significant, tests with an unknown change point (e.g., the Andrews (1993) sup-Wald test) could locate a data-determined break elsewhere. The strength of the result—a p-value of 0.0004 at the imposed date—makes it unlikely that the qualitative conclusion is an artifact of the split, but the precise magnitude of the coefficient change is conditional on the chosen date.

The 2020 pandemic outlier. The COVID-19 shock produced unprecedented quarterly swings in both series (unemployment rose by more than nine percentage points in a single quarter). These observations fall in the second sub-period and, given their leverage, may inflate the estimated post-1990 coefficient. That said, Figure 4 shows the model tracks even these extreme quarters well, and the pre-pandemic post-1990 relationship was already substantially steeper than the earlier era.

Measurement and specification. Aggregating monthly unemployment to quarterly averages, using non-annualized quarterly growth, and fixing the HAC truncation lag at $L = 4$ are all defensible but consequential choices; alternative conventions would shift the point estimates modestly. The difference form also abstracts from the level relationship (the output gap) and from asymmetries between expansions and recessions that a richer, state-dependent model would capture (Owyang and Sekhposyan, 2012).

6.3 Directions for future work

Natural extensions include estimating a time-varying-parameter model to trace the continuous evolution of the coefficient rather than imposing a single break; applying an unknown-break-point test to locate the transition endogenously; identifying the causal Okun elasticity within a structural VAR that separates demand from supply shocks (Foroni and Furlanetto, 2022); and testing for asymmetry between recessions and expansions (Owyang and Sekhposyan, 2012). Each would sharpen the interpretation of the reduced-form comovement documented here.

7 Conclusion

Using U.S. quarterly data from FRED (series `UNRATE` and `GDPC1`) over 1960-Q2 to 2023-Q4, this report estimated the difference form of Okun's law with Newey–West HAC-robust inference. The full-sample Okun coefficient is -0.521 (HAC SE 0.151, $R^2 = 0.590$), implying a break-even growth rate of 2.94% annualized (95% CI [2.51, 3.37]). Splitting the sample at 1990 reveals a statistically significant structural break: the coefficient strengthens from −0.259 to −0.714, a change of −0.455 that is significant at the 1% level ($p = 0.0004$). The result is robust to Delta-Method threshold inference and to a distributed-lag specification in which a significant one-quarter-lagged effect makes the long-run coefficient even larger in magnitude. While simultaneity precludes a strictly causal reading, the evidence points clearly to a labor market that has become substantially more sensitive to the pace of economic growth over the past three decades. All data, code, and results accompany this report to ensure full reproducibility.

A Reproducibility and Runnable Code

All results are fully reproducible. Data were retrieved from FRED on 10 July 2026; the exact series, transformations, and package versions are recorded in `data_provenance.json`. The regression-output table is provided as `regression_output.csv`, the structural-break test as `structural_break_test.json`, and the distributed-lag results as `robustness_lagged_model.json`. The two Python scripts below (dependencies: `pandas`, `numpy`, `requests`, `statsmodels`, `scipy`, `matplotlib`) reproduce every number and figure in this report.

A.1 Data acquisition (`01_data_acquisition.py`)

```
import requests, io
import pandas as pd

FRED = "https://fred.stlouisfed.org/graph/fredgraph.csv?id={}"

def fred(series_id):
    r = requests.get(FRED.format(series_id), timeout=60)
    r.raise_for_status()
    df = pd.read_csv(io.StringIO(r.text))
    df.columns = ["Date", series_id]
    df["Date"] = pd.to_datetime(df["Date"])
    df[series_id] = pd.to_numeric(df[series_id], errors="coerce")
    return df

# UNRATE (monthly) -> quarterly average; GDPC1 already quarterly
unrate = fred("UNRATE")
gdpc1 = fred("GDPC1")

unrate_q = (unrate.set_index("Date").resample("QS").mean().reset_index())
df = pd.merge(unrate_q, gdpc1, on="Date", how="inner")
df = df[(df["Date"] >= "1960-01-01") & (df["Date"] <= "2023-12-31")]

df["Delta_Unemployment"] = df["UNRATE"].diff()
df["GDP_Growth"] = df["GDPC1"].pct_change() * 100.0
df = df.dropna().reset_index(drop=True) # -> 255 quarters, 1960-Q2..2023-Q4
df.to_csv("results/cleaned_okun_data.csv", index=False)
```

Listing 1: Downloading UNRATE and GDPC1 from FRED, aligning to quarterly frequency, and constructing the regression variables.

A.2 Estimation, structural-break test, and robustness (`02_okun_estimation.py`)

```
import numpy as np, pandas as pd, statsmodels.api as sm
from scipy import stats

df = pd.read_csv("results/cleaned_okun_data.csv", parse_dates=["Date"])
Z = stats.norm.ppf(0.975) # 1.959964

def fit(d):
    y = d["Delta_Unemployment"].to_numpy()
```

```
    X = sm.add_constant(d["GDP_Growth"].to_numpy())
    res = sm.OLS(y, X).fit(cov_type="HAC", cov_kwds={"maxlags": 4})
    a, b = res.params
    cov = np.asarray(res.cov_params()) # 2x2 HAC covariance
    gstar = -a / b # threshold G* = -alpha/beta
    grad = np.array([-1.0 / b, a / b**2]) # Delta-Method gradient
    se = float(np.sqrt(grad @ cov @ grad)) # SE of G* (quarterly)
    ci = (gstar - Z*se, gstar + Z*se)
    return dict(alpha=a, beta=b, se_beta=res.bse[1], r2=res.rsquared,
                gstar_q=gstar, gstar_se=se, gstar_ci=ci, n=int(res.nobs))

full = fit(df)
sp1 = fit(df[df["Date"] < "1991-01-01"])
sp2 = fit(df[df["Date"] >= "1991-01-01"])

# ---- Structural-break test: interaction-dummy (Chow) specification ----
D = (df["Date"] >= "1991-01-01").astype(float).to_numpy()
G = df["GDP_Growth"].to_numpy(); y = df["Delta_Unemployment"].to_numpy()
Xi = np.column_stack([np.ones_like(G), D, G, G*D]) # const, D, G, G:D
ir = sm.OLS(y, Xi).fit(cov_type="HAC", cov_kwds={"maxlags": 4})
beta1 = ir.params[3] # slope change
print("beta1 =", beta1, "SE =", ir.bse[3], "p =", ir.pvalues[3])

# ---- Distributed-lag robustness: dU = a + b0 G_t + b1 G_{t-1} ----
def fit_lag(d):
    d = d.sort_values("Date").copy()
    d["G_lag1"] = d["GDP_Growth"].shift(1)
    d = d.dropna(subset=["G_lag1"])
    X = np.column_stack([np.ones(len(d)), d["GDP_Growth"], d["G_lag1"]])
    res = sm.OLS(d["Delta_Unemployment"], X).fit(
        cov_type="HAC", cov_kwds={"maxlags": 4})
    b0, b1 = res.params[1], res.params[2]
    cov = np.asarray(res.cov_params())
    var_lr = cov[1,1] + cov[2,2] + 2*cov[1,2] # Var(b0+b1)
    return b0 + b1, float(np.sqrt(var_lr)) # long-run coef & SE
```

Listing 2: Core estimation: OLS with Newey–West HAC standard errors, the Delta Method for the threshold growth rate, the interaction-dummy structural-break test, and the distributed-lag specification.

B Regression-Output Table (CSV)

The complete machine-readable regression output accompanying this report is `regression_output.csv`. Its contents (baseline models with Delta-Method threshold confidence intervals) are reproduced in Table 5.

Table 5: Contents of `regression_output.csv` (values rounded for display).

Field	Full sample	1960–1990	1991–2023
`Intercept_alpha`	0.382794	0.230263	0.439611
`Intercept_HAC_SE`	0.123391	0.051047	0.102909
`Okun_beta`	−0.520564	−0.258959	−0.713784
`Okun_HAC_SE`	0.150577	0.039008	0.122810
`R_squared`	0.589570	0.490817	0.736152
`Threshold_growth_quarterly_pct`	0.735345	0.889184	0.615888
`Threshold_quarterly_CI95_low`	0.627771	0.673978	0.497963
`Threshold_quarterly_CI95_high`	0.842918	1.104391	0.733814
`Threshold_growth_annualized_pct`	2.941380	3.556738	2.463553
`Threshold_annualized_CI95_low`	2.511086	2.695911	1.991852
`Threshold_annualized_CI95_high`	3.371674	4.417564	2.935254
`N_obs`	255	123	132

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